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BALLISTICS OF SHELLS CONTAINING A VISCOELASTIC FLUID.(U)
JUN 82 R S RIVLIN   DAAG29-79-C-0058
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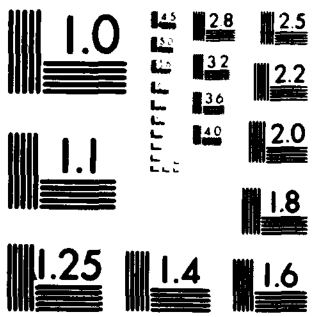

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Work has been carried out on the problems of run-up and spin-up in a viscoelastic fluid. Three simple problems have been considered. These are: (i) The fluid is confined between two infinite rigid parallel plates. These are simultaneously and instantaneously given equal velocities, which are then maintained constant. The resulting velocity field of the fluid is calculated as a function of time. (ii) The fluid is contained in a rigid circular cylinder of infinite length. At some instant the cylinder is given a longitudinal velocity which is

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20. ABSTRACT CONTINUED

subsequently held constant. The resulting velocity field in the fluid is calculated as a function of time. (iii) The rigid containing cylinder is, at some instant, given an angular velocity which is subsequently held constant. The resulting angular velocity field in the fluid is calculated.

FINAL REPORT

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Title: Ballistics of Shells Containing a Viscoelastic Fluid

Personnel Supported: J.Y. Kazakia
R.S. Rivlin

Report Prepared By: R.S. Rivlin

PROGRESS REPORT

Work has been carried out on the problems of run-up and spin-up in a viscoelastic fluid. Three simple problems have been considered. These are:

(i) The fluid is confined between two infinite rigid parallel plates. These are simultaneously and instantaneously given equal velocities, which are then maintained constant. The resulting velocity field of the fluid is calculated as a function of time.

(ii) The fluid is contained in a rigid circular cylinder of infinite length. At some instant the cylinder is given a longitudinal velocity which is subsequently held constant. The resulting velocity field in the fluid is calculated as a function of time.

(iii) The rigid containing cylinder is, at some instant, given an angular velocity which is subsequently held constant. The resulting angular velocity field in the fluid is calculated.

The results obtained on these three problems are contained in three papers [1,2,3].

In all of these papers, the fluid is considered to be isotropic and incompressible and the constitutive assumption is made that in time-dependent simple shearing flows, the shear stress σ, measured at time t, depends linearly on the history $\kappa(\tau)$ $(-\infty < \tau \leq t)$ of the velocity gradient, thus:

$$\sigma = \int_{-\infty}^{t} f(t-\tau)\kappa(\tau)d\tau \; . \tag{1}$$

For a fluid which has N relaxation times $f(t-\tau)$ is given by

$$f(t-\tau) = \eta_0\delta(t-\tau) + \sum_{n=1}^{N} \frac{\eta_n}{\lambda_n} e^{-(t-\tau)/\lambda_n} , \tag{2}$$

where the η's and λ's are positive constants and $\delta(\ )$ is the Dirac delta function. If the fluid has instantaneous elastic response, $\eta_0 = 0$.

The character of the run-up or spin-up behavior depends significantly on whether the fluid does or does not have instantaneous elastic response, i.e. on whether $\eta_0 = 0$ or $\eta_0 \neq 0$. However, the behavior in the former case may be regarded as a limiting case of that in the latter.

The procedure adopted in analyzing each of the problems considered is to set up the equations governing the problem. The Laplace transforms of the equations are formed and solved for the Laplace transform $\bar{v}$ of the velocity. The velocity field v is then obtained by evaluating the inverse Laplace transform.

In the case of run-up between parallel plates, $\bar{v}$ is given by

$$\bar{v} = V \frac{\cosh\{[\rho s/\bar{f}(s)]^{\frac{1}{2}} z\}}{s \cosh\{[\rho s/\bar{f}(s)]^{\frac{1}{2}} h\}} , \tag{3}$$

where $2h$ is the distance between the plates, z is the distance of a generic point of the fluid from the mid-plane, ρ is the density of the fluid, V is the velocity of the plates, and $\bar{f}(s)$ is the Laplace transform of $f(t-\tau)$. v is then given by the inversion integral:

$$v = \frac{1}{2\pi\imath}\int_{\gamma-\imath\infty}^{\gamma+\imath\infty} e^{st}\bar{v}(s,z)ds \ , \tag{4}$$

where γ is an arbitrary positive constant.

In [1] this integration was achieved by using the Residue Theorem. The calculations were carried out in detail, and curves showing the velocity profiles at various times were plotted, for the cases when, in (2), $N = 1$ and $\eta_0 = 0$ (Maxwellian fluid) and $\eta_0 \neq 0$. The expressions for v were obtained in the form of an infinite series, the various terms in which are contributed by the residues at the poles of the integrand in (4).

It was found in [1] that if the fluid is Maxwellian, so that $f(t-\tau)$ may be written as

$$f(t-\tau) = \frac{\eta}{\lambda} e^{-(t-\tau)/\lambda} \ , \tag{5}$$

velocity discontinuities propagate into the fluid from the boundaries and are reflected back and forth. The speed with which these discontinuities propagate is $(\eta/\lambda\rho)^{\frac{1}{2}}$. As the discontinuities propagate, they decay exponentially with decay constant $\frac{1}{2}(\rho/\lambda\eta)^{\frac{1}{2}}$. In the case when the fluid has a single relaxation time, but does not possess instantaneous elasticity, so that $f(t-\tau)$ is given by

$$f(t-\tau) = \eta_0\delta(t-\tau) + \frac{\eta}{\lambda} e^{-(t-\tau)/\lambda} \ , \tag{6}$$

run-up is achieved by a process which is essentially diffusive in character. However, the "steepness" of the front increases as

η_0 decreases and, in the limit $\eta_0 = 0$, becomes a discontinuity.

This interpretation of the velocity field is seen by expressing $\bar{v}$ in (3) in the form

$$\bar{v} = \frac{V}{s} \sum_{k=0}^{\infty} (-1)^k \{\exp[-(2k+1-y)w] + \exp[-(2k+1+y)w]\} , \quad (7)$$

where

$$w = h[\rho s/\bar{f}(s)]^{\frac{1}{2}} , \quad y = z/h . \quad (8)$$

Then,

$$v = V \sum_{k=0}^{\infty} (-1)^k [\Phi_k(y,t) + \Psi_k(y,t)] , \quad (9)$$

where Φ_k and Ψ_k are the inverse Laplace transforms of $\bar{\Phi}_k$ and $\bar{\Psi}_k$ defined by

$$\begin{aligned} \bar{\Phi}_k(y,s) &= \frac{1}{s} \exp[-(2k+1-y)w] , \\ \bar{\Psi}_k(y,s) &= \frac{1}{s} \exp[-(2k+1+y)w] . \end{aligned} \quad (10)$$

In the case when the fluid is Maxwellian, Φ_k and Ψ_k have been evaluated by using Cauchy's theorem with a modified Bromwich contour [2]. The expressions for Φ_k and Ψ_k were obtained in the form of real quadratures. In the more general cases when $f(t-\tau)$ is given by (2) with $\eta_0 = 0$ or $\eta_0 \neq 0$, corresponding results have been obtained by using Cauchy's theorem with a contour which consists of a strip (excluding the origin) in the real half of the complex plane bounded by $\mathrm{Re} s = \gamma$ and

Res = 0 [3].

A procedure similar to that employed in [1] in the case of run-up between parallel plates has been used to solve the problems of run-up and spin-up in an infinite circular cylinder. In each case the expression for the velocity field is obtained as an infinite series. As in the case of run-up between parallel plates the velocity field may, in each case, be interpreted as the superposition of disturbances which propagate from the cylindrical boundary, come to a focus on the axis of the cylinder, then diverge, and are reflected back from points on the opposite ends of the diameters from which they originated.

The methods applied in [3] to the problem of run-up between parallel plates can also be applied to these problems to give alternative expressions for the velocity field. However, this work has not, so far, been completed.

Work has also been carried out on the spin-up of a fluid contained in the region between two parallel infinite discs. The discs are given, at some instant, an angular velocity about a common axis which is thereafter held constant. The constitutive equation adopted for the fluid is a second-order equation of the integral type and it is assumed that the velocities are sufficiently small so that the terms of higher degree than the second in them can be neglected. It is also assumed that the relation of the shear stress to the velocity gradient history, for simple shearing flows, is Maxwellian. It has been found that angular velocity discontinuities propagate into the fluid, similar to the velocity of longitudinal run-up between parallel planes. Behind these discontinuities a secondary flow takes place in axial planes which is radially outwards just behind the front and inwards near the discs.

Work is also in progress on small Rossby number spin-up or spin-down in a fully spun-up viscoelastic fluid contained between parallel discs.

References

[1] Run-up and spin-up in a viscoelastic fluid. I, by J.Y. Kazakia and R.S. Rivlin. Rheologica Acta 20, 111-127 (1981).

[2] Run-up and spin-up in a viscoelastic fluid. II, by R.S. Rivlin. Rheologica Acta (in the press).

[3] Run-up and spin-up in a viscoelastic fluid. III, by R.S. Rivlin. Rheologica Acta (in the press).

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